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Quasi-Antiorder Relational System

Duško Jojić

Department of Mathematics and Informatics
Banja Luka University, 2, Mladen Stojanović Street
78000 Banja Luka, Bosnia and Herzegovina
ducci68@blic.net

Daniel A. Romano

Department of Mathematics and Informatics
Banja Luka University, 2, Mladen Stojanović Street
78000 Banja Luka, Bosnia and Herzegovina
bato49@hotmail.com

Abstract

This paper is within the Bishop's constructive mathematics. A quasi-antiorder relational system means a pair (A, R) where $(A, =, \neq)$ is a set with apartness and R is a consistent and cotransitive binary relation on A. We define and study a quotient system mapping φ such that the factor relation $R/Coker\varphi$ on the factor set $A/Coker\varphi$ is also a quasi-antiorder relational system.

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1 Introduction

This paper is a continuation of the second author's recent papers [12] and [13]. Our setting is Bishop's constructive mathematics ([1], [2], [6], [14]).

The concept of a relational system was introduced by A.I.Maltsev ([4], [5]) We will restrict our consideration to relational systems with only one binary relation. Hence, by a relational system we will take a pair $\boldsymbol{A} = (A, R)$, where $(A, =, \neq)$ is a set with apartness and $R \subseteq A \times A$, i. e., R is a binary relation on A. Relational systems play an important role both in mathematics and in applications since every formal description of a real system can be done by means of relations. For these considerations we often ask about a certain factorization of a relational system $\boldsymbol{A} = (A, R)$ because it enables us to introduce the method of abstraction on $\boldsymbol{A}$. Hence, if θ is a coequality on A, we ask about a 'factor relation' R/θ on the factor set A/θ such that the factor system $(A/\theta, R/\theta)$ shares some of 'good' properties of $\boldsymbol{A}$.

In this paper, we are mostly interested in relational systems $\boldsymbol{A} = (A, R)$ where R is consistent, i.e. $(\forall x, y \in A)((x, y) \in R \Longrightarrow x \neq y)$ and cotransitive, i.e. $(a, c) \in R$ imply $(\forall b \in A)((a, b) \in R \vee (b, c) \in R)$. In that case, A is called a *consistent and cotransitive system* or a *quasi-antiorder system*.
Our intention is to study the situation on $\boldsymbol{A}$ such that the system $(A/\theta, R/\theta)$ is also consistent and cotransitive.

Let us note that a similar task for anti-ordered sets was already studied in [9]-[13], and we will try to modify that construction for consistent and cotransitive relational systems. According to [9] and [10], if $(S, =, \neq, \cdot, \alpha)$ is an anti-ordered semigroup and σ a quasi-antiorder on S, then the relation q on S, defined by $q = \sigma \cup \sigma^{-1}$, is an anticongruence on S and the set S/q is an anti-ordered semigroup under anti-order Θ defined by $(xq, yq) \in \Theta \Longleftrightarrow (x, y) \in \sigma$. An anticongruence q on S is called *regular* if there is an antiorder "θ_1" on S/q satisfying the following conditions:
(1) $(S/q, =_1, \neq_1, \cdot, \theta_1)$ is an anti-ordered semigroup;
(2) The mapping $\pi : S \ni a \longmapsto aq \in S/q$ is an anti-order reverse isotone epimorphism.
We call the antiorder "θ_1" on S/q a regular antiorder with respect to a regular anticongruence q on S and the anti-order α .

In this article we are interested in quasi-antiorder systems $((A, =, \neq), \alpha)$ and θ coequality on A such that the relation α/θ, defined by $(a\theta, b\theta) \in \alpha/\theta \Longleftrightarrow (a, b) \in \theta^C \circ \alpha \circ \theta^C$, is a quasi-antiorder relation on A/θ . The system $\boldsymbol{A}/\theta = (A/\theta, \alpha/\theta)$ will be called a *quotient system of* A by θ.

2 Preliminaries

Let $(A, =, \neq)$ be a set in the sense of books [1], [2], [6] and [14], where "$\neq$" is a binary relation on A which satisfies the following properties:

$$\neg(x \neq x),\ x \neq y \Longrightarrow y \neq x,\ x \neq z \Longrightarrow x \neq y \vee y \neq z,$$
$$x \neq y \wedge y = z \Longrightarrow x \neq z,$$

called *apartness* (A. Heyting). The apartness is *tight* (W. Ruitenburg) if $\neg(x \neq y) \Longrightarrow x = y$ holds. Let Y be a subset of A and $x \in A$. The subset Y of A is *strongly extensional* in A if and only if $y \in Y \Longrightarrow y \neq x \vee x \in Y$ ([1], [8]). If

$x \in A$, we defined ([8]) $x \bowtie Y$ by $(\forall y \in Y)(y \neq x)$.

Let $f : (A, =, \neq) \longrightarrow (B, =, \neq)$ be a function. We say that it is:
(a) f is *strongly extensional* if and only if $(\forall a, b \in A)(f(a) \neq f(b) \Longrightarrow a \neq b)$;
(b) f is an *embedding* if and only if $(\forall a, b \in A)(a \neq b \Longrightarrow f(a) \neq f(b))$.
Let $\alpha \subseteq A \times B$ and $\beta \subseteq B \times C$ be relations. The *filled product* ([7]) of relations α and β is the relation

$$\beta * \alpha = \{(a, c) \in A \times C : (\forall b \in B)((a, b) \in \alpha \vee (b, c) \in \beta\}.$$

A relation $q \subseteq A \times A$ is a *coequality* relation on A if and only if holds:

$$q \subseteq \neq, \ q = q^{-1}, \ q \subseteq q * q.$$

If q is a coequality relation on set $(A, =, \neq)$, we can construct factor-set $(A/q, =_1, \neq_1)$ with

$$aq =_1 bq \Longleftrightarrow (a, b) \bowtie q, \ aq \neq_1 bq \Longleftrightarrow (a, b) \in q.$$

A relation α on A is an *antiorder* ([9]-[12]) on A if and only if

$$\alpha \subseteq \neq, \ \alpha \subseteq \alpha * \alpha, \ \neq \subseteq \alpha \cup \alpha^{-1}, \ (\alpha \cap \alpha^{-1} = \emptyset).$$

As in [9], a relation $\tau \subseteq A \times A$ is a *quasi-antiorder* on A if and only if

$$\tau \subseteq (\alpha \subseteq) \neq, \ \tau \subseteq \tau * \tau, \ (\tau \cap \tau^{-1} = \emptyset).$$

Let $f : ((A, =, \neq), \alpha) \longrightarrow ((B, =, \neq), \beta)$ be a strongly extensional function of relational systems. f is called *isotone* if $(\forall x, y \in A)((x, y) \in \alpha \Longrightarrow (f(x), f(y)) \in \beta)$; f is called *reverse isotone* if and only if $(\forall x, y \in A)((f(x), f(y)) \in \beta \Longrightarrow (x, y) \in \alpha)$. The strongly extensional mapping f is called an isomorphism if it is injective and embedding, onto, isotone and reverse isotone. A and B called isomorphic, in symbol $A \cong B$, if exists an isomorphism between them.

Let us note if $\varphi : ((A, =, \neq), \alpha) \longrightarrow ((B, =, \neq), \beta)$ is a strongly extensional of quasi-antiorder systems, then $\varphi^{-1}(\beta)$ is a quasi-antiorder included in α .

3 Results

Firstly, we will start with two definitions:

Definition 1 Let $((A, =, \neq), \alpha)$ and $((B, =, \neq), \beta)$ be two relational systems. A reverse isotone strongly extensional mapping $\varphi : A \longrightarrow B$ is called a *quotient system* mapping (abbreviated to QS-mapping) of A to B if holds

$$\alpha \subseteq Ker\varphi \circ \varphi^{-1}(\beta) \circ Ker\varphi.$$

In the case when φ is onto, B is called a *quotient set* of A by φ .

Note: It is easy to see that QS-mapping of two quasi-antiorder relational systems is isotone mapping. In fact, if (x, y) is an arbitrary element of α, then there exist elements a, b of A such that $\varphi(x) = \varphi(a)$ and $(\varphi(a), \varphi(b)) \in \beta$ and $\varphi(b) = \varphi(y)$. Since β is a quasi-antiorder relation on B, out of $(\varphi(a), \varphi(x)) \in \beta \subseteq \neq$ or $(\varphi(x), \varphi(y)) \in \beta$ or $(\varphi(y), \varphi(b)) \in \beta \subseteq \neq$ we have $(\varphi(x), \varphi(y)) \in \beta$. So, the QS - mapping is a isotone mapping.

Definition 2 Let $((A, =, \neq), \alpha)$ be a relational systems and θ be a coequality relation on A. Define a binary relation α/θ on the set A/θ as follows:

$$(a\theta, b\theta) \in \alpha/\theta \Longleftrightarrow (a, b) \in \theta^C \circ \alpha \circ \theta^C .$$

The system $\boldsymbol{A}/\theta = (A/\theta, \alpha/\theta)$ will be called a quotient system of $\boldsymbol{A}$ by θ .

The following statement is obvious.

Lemma 1 *Let* $\mathbf{A} = (A, \alpha)$ *be a relational system and* θ *be a coequality on* A.
(i) *If* $\alpha \subseteq \theta$, *then* α/θ *is consistent.*
(ii) α/θ *is symmetric if and only if* α *is symmetric.*
(iii) *If* α *is cotransitive, then* α/θ *is cotransitive too.*

Proof: (1) Suppose that α is included in θ and let $(a\theta, b\theta)$ be an arbitrary element of α/θ. Then $(a, b) \in \theta^C \circ \alpha \circ \theta^C \subseteq \theta^C \circ \theta \circ \theta^C \subseteq \theta$. Thus, $a\theta \neq_1 b\theta$.
(2) Immediately follows from definitions of symmetry and α/θ .
(3) Let $(a\theta, c\theta)$ be an element of α/θ and let $b\theta$ is an arbitrary element of A/θ. Then $(a, c) \in \theta^C \circ \alpha \circ \theta^C$, i.e. then there exist elements x, z in A such that

$$(a, x) \bowtie \theta \ \wedge \ (x, z) \in \alpha \ \wedge \ (z, c) \bowtie \theta.$$

Hence,

$$(a, x) \bowtie \theta \ \wedge \ (\forall y \in A)((x, y) \in \alpha \vee (y, z) \in \alpha) \ \wedge \ (z, c) \bowtie \theta$$

and

$$(\forall y \in A)(((a, x) \bowtie \theta \wedge (x, y) \in \alpha \wedge (z, c) \bowtie \theta) \vee ((a, x) \bowtie \theta \wedge (y, z) \in \alpha \wedge (z, c) \bowtie \theta)) \ .$$

Thus, for $y = b$, out of above formula we have

$$((a, x) \bowtie \theta \wedge (x, b) \in \alpha \wedge (b, b) \bowtie \theta) \vee ((b, b) \bowtie \theta \wedge (b, z) \in \alpha \wedge (z, c) \bowtie \theta))$$

and, finally

$$(a, b) \in \theta^C \circ \alpha \circ \theta^C \vee (b, c) \in \theta^C \circ \alpha \circ \theta^C . \ \square$$

In the following assertion we give an answer on on the question: If $\boldsymbol{A} = (A, \alpha)$ is a quasi-antiorder relational system and θ is a coequality relation on A, when the factor-system $(A/\theta, \alpha/\theta)$ is a quasi-antiorder relational system too?

Corollary 1.1. *Let* $\mathbf{A} = (A, \alpha)$ *be a consistent and cotransitive relational system and* θ *be a coequality on* A *such that* $\alpha \subseteq \theta$. *Then, the relation* α/θ *is a consistent and cotransitive on set* A/θ .

However, following [3] the lower and upper bounds can be defined also for general relational systems. Let $\boldsymbol{A} = (A, \alpha)$ be a relational system and a, b be elements of A. In the tradition of [3] but some different to it we introduce the following notations:

$$L_A(a, b) = \{x \in A : (x, a) \in \alpha \vee (x, b) \in \alpha\}$$
$$U_A(a, b) = \{y \in A : (a, y) \in \alpha \vee (b, y) \in \alpha\}.$$

If $a = b$, we will write $L_A(a)$ and $U_A(b)$ instead αa and $b\alpha$ respectively. Clearly, if α is a consistent relation, then $a \bowtie L_A(a)$ and $a \bowtie U_A(a)$ for each $a \in A$. It is easy to prove the following two assertions:

Remark. Let $\boldsymbol{A} = ((A, =, \neq), \alpha)$ be a quasi-antiorder relational systems. Then:

$$(a, b) \bowtie \alpha \text{ iff } L_A(a, b) = L_A(a) \text{ iff } U_A(a, b) = U_A(b)$$

and

$$(a, b) \in \alpha \Longleftrightarrow A = U_A(a) \cup L_B(b).$$

Proof. (1) It is clear that $L_A(a) \subseteq L_A(a, b)$. Suppose that $(a, b) \bowtie \alpha$. Now out of $(x, b) \in \alpha$ and $(a, b) \bowtie \alpha$ follows $(x, a) \in \alpha$. Therefore, $(a, b) \bowtie \alpha \Longrightarrow L_A(a) = L_A(a, b)$.

(2) Let $L_A(a, b) = L_A(a)$ holds. Out of $a \bowtie L_A(a) = L_A(a, b)$ we conclude that $(a, b) \bowtie \alpha$. In fact, let (u, v) be an arbitrary element of α. Then, we have $(u, a) \in \alpha$ or $(a, b) \in \alpha$ or $(b, v) \in \alpha$. Suppose that $(a, b) \in \alpha$. Then, it would be

$$a \in L_A(b) \subseteq L_A(a) \cup L_A(b) = L_A(a, b) = L_A(a).$$

Since the least is impossible, we conclude that $(a, b) \neq (u, v)$.

(3) The proof for $(a, b) \bowtie \alpha$ iff $U_A(a, b) = U_A(b)$ we get analogously. ♦

Following definition of LU-mapping in [3] we introduce analogous notion:

Definition 3. Let $\boldsymbol{A} = ((A, =, \neq), \alpha)$ and $\boldsymbol{B} = ((B, =, \neq), \beta)$ be two relational systems. A surjective strongly extensional mapping $f : A \longrightarrow B$ is called an LU-*mapping* if

$$f(L_A(x,y)) = L_A(f(x), f(y)) \text{ and } f(U_A(x,y)) = U_A(f(x), f(y))$$

holds for all $x, y \in A$.

If $\boldsymbol{A} = ((A, =, \neq), \alpha)$ and $\boldsymbol{B} = ((B, =, \neq), \beta)$ are quasi-antiorder relational systems and if f is a strongly extensional and reverse isotone surjective mapping, then

$$L_B(f(x), f(y)) \subseteq f(L_A(x,y)) \text{ and}$$

$$U_B(f(x), f(y)) \subseteq f(U_A(x,y)).$$

Indeed, let $z \in L_B(f(x), f(y))$, i.e. let $(z, f(x)) \in \beta$ or $(z, f(y)) \in \beta$. Then there exists an element t of A such that $z = f(t)$ and $(f(t), f(x)) \in \beta$ or $(f(t), f(y)) \in \beta$. Since, f is a reverse isotone mapping, we have $(t, x) \in \alpha$ or $(t, y) \in \alpha$. Thus, $t \in L_A(x, y)$ and $z = f(t) \in f(L_A(x, y))$. Proof for inclusion $U_B(f(x), f(y)) \subseteq f(U_A(x, y))$ is analogous.

In the following theorem we prove that every strongly extensional reverse isotone QS-mapping between two quasi-antiorder relational systems is LU-mapping.

Theorem 1. *Let* $\mathbf{A} = ((A, =, \neq), \alpha)$ *and* $\mathbf{B} = ((B, =, \neq), \beta)$ *be quasi-antiorder relational systems and* $f : A \longrightarrow B$ *be a strongly extensional reverse isotone surjective QS-mapping. Then* f *is a strongly extensional isotone and reverse isotone LU-mapping.*

Proof. (1) Let z be an arbitrary element of $f(L_A(x, y))$. Then there exists an element t of $L_A(x, y)$ such that $z = f(t)$ and $(t, x) \in \alpha \vee (t, y) \in \alpha$. Since f is QS-mapping, then there exist elements $a, b, c, d \in A$ such that

$$z = f(t) = f(a) \wedge (f(a), f(b)) \in \beta \wedge f(b) = f(x)$$

or

$$z = f(t) = f(c) \wedge (f(c), f(d)) \in \beta \wedge f(d) = f(y).$$

Thus, we have $z \in \{u \in B : (u, f(x)) \in \beta \vee (u, f(y)) \in \beta\} = L_B(f(x), f(y))$.
(2) The second inclusion $f(U_A(x, y)) \subseteq U_B(f(x), f(y))$ we prove analogously. □

The first important result about relational system $\boldsymbol{A}/\theta$ is given by the following theorem.

Theorem 2 *Let* $\mathbf{A} = ((A, =, \neq), \alpha)$ *be a quasi-antiorder relational systems. If* θ *is a coequality on* A *such that* $\alpha \subseteq \theta$*, then the canonical mapping* $\pi : A \longrightarrow A/\theta$ *is a QS-mapping.*

Proof: Let θ be a coequality relation on a quasi-antiorder relational systems $((A,=,\neq),\alpha)$ and let α/θ be a relation on A/θ defined by

$$(a\theta, b\theta) \in \alpha/\theta \Longleftrightarrow (a,b) \in \theta^C \circ \alpha \circ \theta^C$$

For the mapping $\pi : A \longrightarrow A/\theta$, defined by $\pi(a) = a\theta$ $(a \in A)$, we have:
(i) For elements a and b of A such that $a = b$ we have $(a,b) \bowtie \theta \supseteq \theta^C \circ \alpha \circ \theta^C$. Hence, if (u,v) be an arbitrary element of $\theta^C \circ \alpha \circ \theta^C$, then we have $(a,b) \neq (u,v) \in \theta^C \circ \alpha \circ \theta^C$. So, finally, we have $a\theta =_1 b\theta$.
(ii) It is obvious that is a strongly extensional mapping.
(iii) By Corollary 1.1, the relation α/θ is a quasi-antiorder and the system $\boldsymbol{A}/\theta$ is quasi-antiorder system. Suppose that $(a\theta, b\theta) \in \alpha/\theta$, i.e. suppose that there exists elements x, y of A such that $(a,x) \bowtie \theta$ and $(x,y) \in \alpha$ and $(y,b) \bowtie \theta$. Since $\theta \supseteq \alpha$, out of $(x,a) \in \alpha \vee (a,b) \in \alpha \vee (b,y) \in \alpha$ we have $(a,b) \in \alpha$ because $(x,a) \bowtie \alpha$ and $(b,y) \bowtie \alpha$. Hence, from $(\pi(a), \pi(b)) \in \alpha/\theta$ we conclude that $(a,b) \in \alpha$. Therefore, the mapping π is reverse isotone.
(iv) If $(x,y) \in \alpha$, then $(x,y) \in \theta^C \circ \alpha \circ \theta^C$, i.e. then $(x\theta, y\theta) \in \alpha/\theta$. Thus, $(x,y) \in \pi^{-1}(\alpha/\theta)$. Therefore, we have $\alpha \subseteq \pi^{-1}(\alpha/\theta) \subseteq \theta^C \circ \pi^{-1}(\alpha/\theta) \circ \theta^C$. So, the mapping π is a QS-mapping. □

The second main result about the relational system $\boldsymbol{A}/\theta$ is given by the following theorem and corollary.

Theorem 3 *Let* $\mathbf{A} = ((A,=,\neq),\alpha)$ *and* $\mathbf{B} = ((B,=,\neq),\beta)$ *be quasi-antiorder relational systems and let* $\varphi : A \longrightarrow B$ *be a surjective QS-mapping . Then* $Coker\varphi$ *is a coequality on* $\mathbf{A}$ *and there exists the strongly extensional and embedding isotone and reverse isotone bijective mapping* $\psi : \mathbf{A}/Coker \longrightarrow ((B,\neg\neq,\neq),\beta)$.
Proof: We will verify first that $\psi : A/Coker\varphi \longrightarrow B$, defined by $\psi(aq) = \varphi(a)$, where $q = Coker\varphi$, is a strongly extensional QS-mapping of sets such that $\psi \circ \pi = \varphi$. Let α/q be a quasi-antiorder on set A/q. Then:
(1) The relation $\psi : A/q \longrightarrow B$, defined by $\psi(aq) = \varphi(a)$, is a strongly extensional and embedding relation:

$$\begin{aligned}\psi(aq) \neq \psi(bq) &\Longleftrightarrow \varphi(a) \neq \varphi(b)\\ &\Longleftrightarrow (a,b) \in Coker\varphi = q\\ &\Longleftrightarrow aq \neq_1 bq.\end{aligned}$$

(2) The ψ is an injective relation: In fact, since $\psi(aq) = \psi(bq)$ is equivalent with $(a,b) \in Ker\varphi$, for arbitrary element $(u,v) \in q$ we have $(a,b) \neq (u,v) \in q$. Thus, $aq =_1 bq$.
(3) Let $aq =_1 bq$, i.e. let $(a,b) \bowtie q$ and suppose that $\psi(aq) \neq \psi(bq)$. Thus, we conclude $(a,b) \in q$ which is impossible. So, should be $\neg(\psi(aq) \neq \psi(bq))$. Therefore, the relation ψ is a strongly extensional, injective and embedding mapping from A onto $(B,\neg\neq,\neq)$.

(4) Let $(\psi(aq), \psi(bq)) \in \beta$, i.e. let $(\varphi(a), \varphi(b)) \in \beta$. Thus $(a, b) \in \alpha \subseteq q^C \circ \alpha \circ q^C$ and $(aq, bq) \in \alpha/q$. So, the mapping ψ is reverse isotone.

(5) Let (aq, bq) be an arbitrary element of α/q, i.e. let $(a, b) \in q^C \circ \alpha \circ q^C$. Since $\alpha \subseteq Ker\varphi \circ \varphi^{-1}(\beta) \circ Ker\varphi$, we have

$$(a, b) \in q^C \circ \alpha \circ q^C \subseteq q^C \circ Ker\varphi \circ \varphi^{-1}(\beta) \circ Ker\varphi \circ q^C.$$

Therefore, there exist elements x, x', y', y of A such that $(a, x) \bowtie q$ and $(x, x') \in Ker\varphi$ and $(x', y') \in \varphi^{-1}(\beta)$ and $(y', y) \in Ker\varphi$ and $(y, b) \bowtie q$, i.e. holds $(a, x) \bowtie q$ and $(x, x') \in Ker\varphi$ and $(x', y') \in \varphi^{-1}(\beta)$ and $(y', y) \in Ker\varphi$ and $(y, b) \bowtie q$. Further on, out of $(\varphi(x'), \varphi(y')) \in \beta$ we have $(\varphi(x'), \varphi(x)) \in \beta \subseteq \neq$ or $(\varphi(x), \varphi(a)) \in \beta \subseteq \neq$ or $(\varphi(a), \varphi(b)) \in \beta$ or $(\varphi(b), \varphi(y)) \in \beta \subseteq \neq$ or $(\varphi(y), \varphi(y')) \in \beta \subseteq \neq$. Finally, we have $(\varphi(a), \varphi(b)) \in \beta$ and the mapping ψ is an isotone mapping. □

Corollary 3.1 *Let* $\mathbf{A} = ((A, =, \neq), \alpha)$ *and* $\mathbf{B} = ((B, =, \neq), \beta)$ *be quasi-antiorder relational systems, where the apartness on* B *are tight, and let* $\varphi : \mathbf{A} \longrightarrow \mathbf{B}$ *be a surjective QS-mapping . Then* $Coker\varphi$ *is a coequality on* $\mathbf{A}$ *and there exists the isomorphism:* $A/Coker\varphi \cong ((B, =, \neq), \beta)$ *as quasi-antiorder relational systems.*

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